Name:_____ Date:_____

**SOLUTION Pre Calculus 12: HW Section 1.5 Combined Transformations**

1. Indicate what the function $y = f(x)$ will become after each transformation in the specified order:

a) 1. Horizontal Shift of 3 units left
x → x + 3
2. Horizontal expansion and reflection by a factor 3
x → (-1/3)x

$HS\ 3L \quad y = f(x+3)$

$HE\ \&\ R \quad y = f\left(-\frac{1}{3}x+3\right)$

b. 1. Horizontal Expansion and reflection by a factor of 3
2. Horizontal Shift of 3 lefts left

$HE\ \&\ R: y = f\left(-\frac{1}{3}x\right)$

$HS3L: y = f\left(-\frac{1}{3}(x+3)\right)$

c) A vertical compression and reflection by a factor of -0.75
y → (4/3)y and y → -y

Vertical shift of 8 units up
y → y – 8

$VC\ VR: \frac{-4y}{3} = f(x)$

$VS: \frac{-4}{3}(y-8) = f(x)$

Then simplify:

$y-8 = -\frac{3}{4}f(x)$

$y = \frac{-3}{4}f(x)+8$

d) Vertical shift of 8 units up and then a vertical compression by a factor of -0.75

$VS: y = f(x)+8$

$V.C: y = -0.75[f(x)+8]$

$y = \frac{-3}{4}f(x)-6$

e) A vertical expansion by a factor of 2 and then a reflection over the x-axis. Then a horizontal compression by a factor of 0.25.

$VE: y \to \frac{1}{2}y: \quad y = 2f(x)$

$VR: y \to -y: \quad y = -2f(x)$

$HC: x \to 4x \quad y = -2f(4x)$

f) A horizontal shift of 3 units left and 2 units up. Then a reflection on both axis. Then a HE of 3 and VC of 0.3.

$3L\ \&\ 2UP: \quad y = f(x+3)+2$

$VR\ \&\ HR: \quad -y = f(-x+3)+2$

$y = -f(-x+3)-2$

$HE\ \&\ VC: \frac{3}{10}y = f\left(-\frac{x}{3}+3\right)-2$

Now simplify:

$y = \frac{10}{3}f\left(-\frac{1}{3}(x-9)\right)-\frac{10}{3}$

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2. When two transformations are performed in different orders, will the resulting function always be the same or always different? Explain:

   This is a difficult one to answer. It depends on the transformations.

   1. If one transformation is horiztonal and the other is vertical, then the order doesn't matter. You can do either the vertical transformation then horizontal transformation. OR the other way around
   2. If both transformations are horizontal OR both are vertical, then it depends on the transformation itself. An expansion/compression with a reflection can be done in any order.
   3. A translation with either a reflection or exp/compression CAN NOT be done in any order. The ORDER of which these transformations are important and will affect the resulting equation.

   For the next few questions, it will gives us examples on how the order of translations with exp/comp can affect your answers.

3. Given the transformation: $y=\sqrt{x} \rightarrow y=\sqrt{2x-6}$ indicate the transformation in order:

Method 1: Factoring method: Factor the 2 inside the root: $y=\sqrt{2(x-3)}$

So the order of our transformation will be: 1. Hor Com by 0.5 2. Hor Shift of 3 right

Method 2: Circle Method: $y=\sqrt{x} \quad \rightarrow \quad y=\sqrt{x-6} \quad \rightarrow y=\sqrt{2x-6}$

i) HS of 6 units right first ii) Horizontal compression by 0.5

READ THIS NEXT PART!!!

The two methods differ in order and the type of translation. The factor method always reads from left to right. So we always Exp/Com/Reflect before we perform any translation. In this case, it's 3 units right.

The circle method will shift first, and it's 6 units right!! Not 3, like the first method. By shifting first, the amount that we translate will be different!!!

4. Given the transformation: $y=x^3 \rightarrow y=3x^3-8$ indicate the transformation in order:

   Vertical expansion by 3 and then 8 units down. $y=3x^3 \rightarrow y=3x^3-8$

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5. Given the transformation: $y = x^3 \rightarrow y = 3(3x-6)^3 - 8$ indicate the transformation in order:

To find the transformation in order, simplify and factor all coefficients of "x" and "y"

$y = x^3 \rightarrow y = 3(3x-6)^3 - 8$

$\rightarrow y = 3(3(x-2))^3 - 8$

$\rightarrow y = 3(3)^3(x-2)^3 - 8$

$\rightarrow y = 81(x-2)^3 - 8$

Several ways to read this:

i) V.E. by 81 then 2. HS of 2 right and then 3. VS 8 down

$y = x^3 \rightarrow y = 3(3x-6)^3 - 8$

$\rightarrow y = 3(3(x-2))^3 - 8$

ii) VE. By 3 2. HC. By 1/3 3. 2 Right 4, 8 down

iii) VE by 3 , HS 6 right , HC by 1/3 , VS 8 down

$y = 3x^3 \rightarrow y = 3(x-6)^3 \rightarrow y = 3(3x-6)^3 \rightarrow y+8 = 3(3x-6)^3$

6. The function $y = \sqrt{x}$ is horizontally expanded by a factor of 4. With what VE/VC will result in the same equation?

If we expand this equation Horizontally by a factor of 4, it will become: $y = \sqrt{\frac{1}{4}x}$

If we manipulate this equation and square root the ¼, it will become: $y = \frac{1}{2}\sqrt{x}$

This is the same as a vertical compression by a factor of 0.5

7. For what factor "K" will the transformation of $y = f(x) \rightarrow ky = f(x)$ transform the function from $y = x^2 \rightarrow y = (4x)^2$ ?

First, let's modify the equation:

$y = x^2 \rightarrow y = (4x)^2$

$\rightarrow y = 16x^2$ Therefore "k" is equal to 1/16

$\rightarrow \frac{1}{16}y = x^2$

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8. What is the transformation required to convert $y=(x-3)^2 \rightarrow y=(4x-12)^2$ ? Name two different sets of solutions:

i)Circle method: 9 units Right and then HC by ¼ $y=(x-3)^2 \rightarrow y=(x-9-3)^2 \rightarrow y=(4x-9-3)^2$

ii) factor method: $y=(4(x-3))^2$ HC by ¼ and then 3 units right

iii) Fancy method for those of you who like things fancy..... $y=16(x-3)^2$ VE by 16 and then 3 units right

9. Indicate all the transformations that is required to change from $y=f(x)$ to the equation give:

| | |
|---|---|
| a) $y=f\left(\frac{2}{3}x-1\right)$ | b) $y=-\frac{2}{3}f(x)+\frac{4}{5}$ |
| c) $y=4f(-2x)$ | d) $y=2f(3y+4)+5$ |
| e) $\frac{1}{2}y=f\left(\frac{1}{3}x-\frac{1}{4}\right)+\frac{1}{5}$ | f) $y=3f\left(\frac{1}{5}(2x-3)\right)$ |

10. Given the four transformations in the given order, what will function $y=f(x)$ result in?

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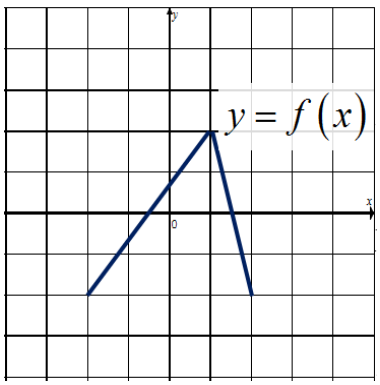
a) 1st) $x \to -\frac{1}{2}x$ 2nd) $y \to \frac{-y}{4}$ 3rd) $x \to x+4$ 4th) $y \to y-12$

b) 1st) $x \to 2-x$ 2nd) $y \to 5-\frac{1}{2}y$ 3rd) $y \leftrightarrow x$ 4th) $y \to y+4$

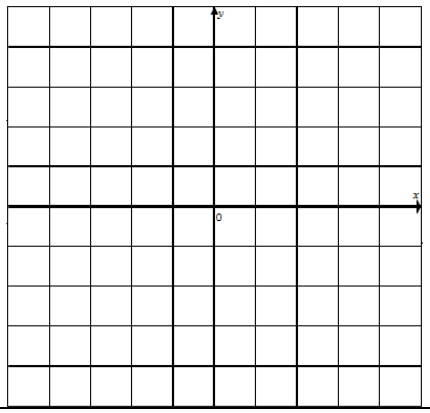
11. Point (e,f) is on the graph of $y = f(x)$, what point must be on the following functions:

| | |
|---|---|
| a) $y = -\frac{1}{4} f(x-3)$ | b) $\frac{-3}{4} y = f(10-4x)+1$ |

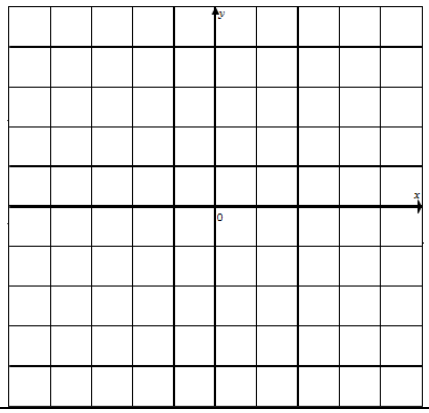
12. Given the graph of $y = f(x)$, draw the graph of the following functions:




a) $y = 2f(2x)$




b) $y = f(2x-1)$

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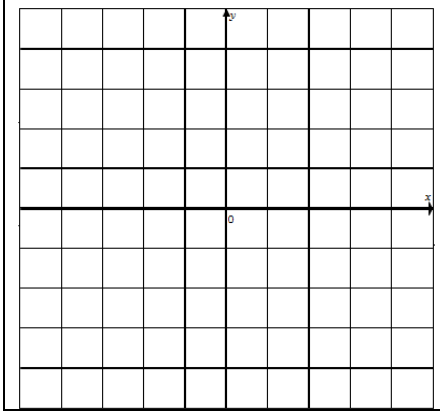
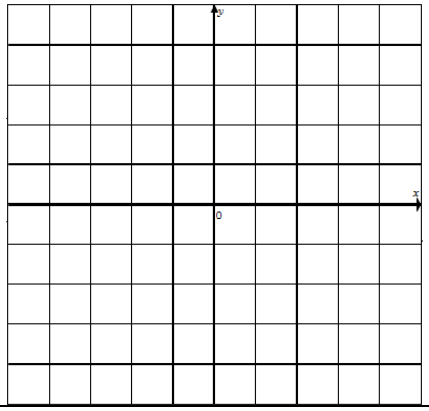
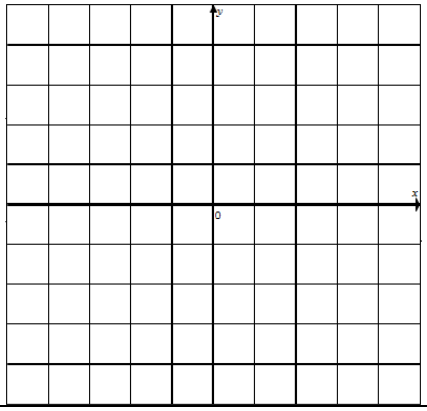
c) $y = 0.5f(x) + 1$



d) $x = f(2x)$

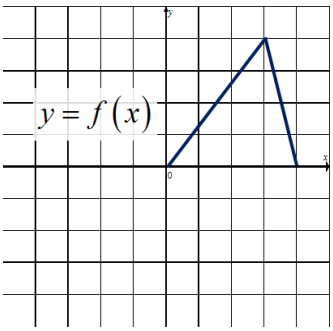


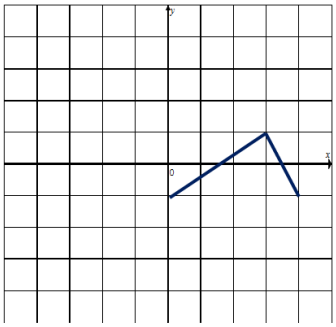
f) $y = 2f(2x-1) - 1$



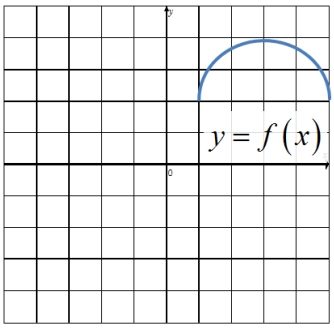
13. Given the graph of $y = f(x)$ on top, what is the equation of the corresponding graph below it:

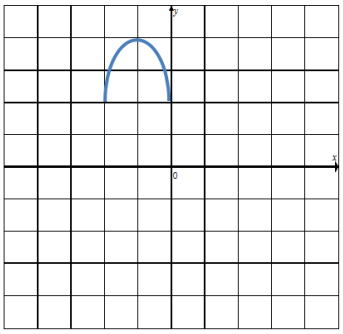
a)



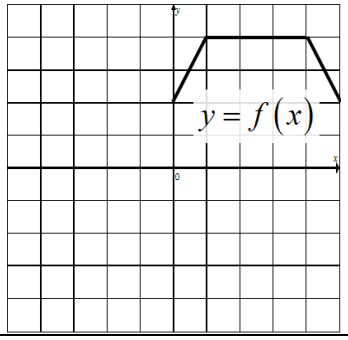



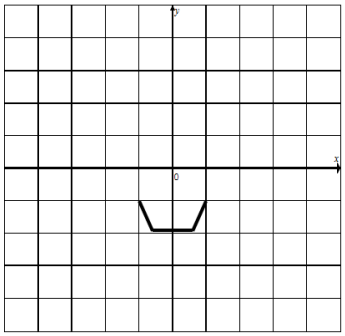
b)



c)



14. Indicate the transformation required to go from the left function to the right. List the transformation in order:

a) $y = \sqrt{x} \rightarrow y = \sqrt{5-3x}$

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b) $y = 3^x \rightarrow y = 4\left(3^{2x+1}\right) - 6$

For this question ,first factor all coefficients of “x’ and “y”

$y = 3^x \rightarrow y = 4\left(3^{2(x+0.5)}\right) - 6$

1. VE. By 4  2. HC. By ½  3. HS 0.5 LEFT  4. VS 6 down

c) $y = \sqrt{x} \rightarrow y = 12\sqrt{-x-12} + 11$

For this question ,first factor all coefficients of “x’ and “y”

d) $y = |2x+1| \rightarrow y = 3\left|\frac{4}{5}x + 12\right| - 1$

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